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Initial Efforts at Asteroid Lightcurve Inversion

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Abstract

The problem of determining the shape of an asteroid from its lightcurve has been studied for many years. Henry Norris Russell presented a paper in 1906 that said it couldn't be done with any certainty. However, further study during the 20th century said otherwise and several methods were developed that had various levels of success. In the last several years, many asteroid shape and spin axis models have been produced using methods pioneered by Mikko Kaasalainen and others. The author has converted the original FORTRAN and C code of Kaasalainen and Durech so that it is available to anyone wanting to develop their own inversion program. Models based on lightcurves the author and others have obtained are shown.

1. Introduction

Imagine an asteroid painted with alternate sections of black and white, such as shown in Figure 1. Now rotate the asteroid and measure the reflected light. Assuming the object reflects light geometrically, this "asteroid" will show no lightcurve variations over its rotation.

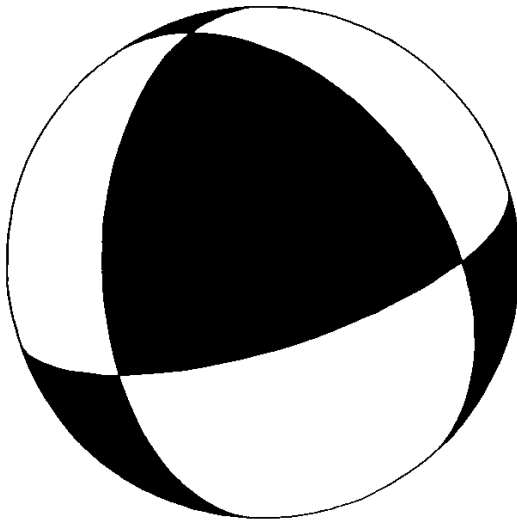


Figure 1. A sphere painted so that it will have a zero-amplitude lightcurve. After Magnusson et al (1989).

This logic of being able to paint (change the albedo) any object in specific ways lead Henry Norris Russell to conclude in his famous 1906 paper that it was very improbable that one could determine the shape of an asteroid from its lightcurve alone. In fact,

one could paint an asteroid so that it was faintest when seen with its largest surface profile and brightest when seen with its smallest profile.

However, despite what would prove to be an incorrect assessment of the chances for success, that paper did contain some important conclusions that proved useful in later studies of the inversion problem, i.e., those concerning albedo variations and scattering law. In particular, he showed that it was theoretically always possible to determine the position of the asteroid's equator.

Eventually, it came to be accepted that asteroids are not painted like checkerboards but are, for the most part, painted a uniform gray, or least to a first order sufficient to find a reasonable model of the spin axis and shape. For a more complete discussion of the history of lightcurve inversion and the various methods that developed prior those of Kaasalainen et al, the reader is referred to the chapter in *Asteroids II* by Magnusson et al.

2. Shape versus Spin Axis

Before going on, it's important to distinguish the differences and importance of what can be learned from an asteroid's lightcurve.

The first is the orientation of the spin axis of the asteroid and its exact rotation rate. In fact, this is really the more important information over the shape of the asteroid, the fact that finding and showing the shape is much more dramatic.

The spin axis is given as the ecliptic coordinates, longitude and latitude, of the direction in which the north pole of the spin axis (+Z) is pointing. If the

latitude is 0° , then the equatorial plane of the asteroid is coincident with the plane of the ecliptic.

If the orbit of the asteroid also has a very small inclination to the ecliptic, then we not see much change in the shape of lightcurve except for those caused by the variations at large phase angles when shadows on the asteroid become longer and change shape.

If the latitude is near 90° , then at some apparitions, we'll see the asteroid nearly pole-on at other others we'll see it "broadside". This allows for even more dramatic changes in the shape and amplitude of the lightcurve at different apparitions.

Why is the spin axis more important? More and more its being accepted that thermal re-radiation of sunlight, the so-called YORP effect, is the primary force behind the formation of binary asteroids for smaller objects and the excess of fast and slow rotators among the MBA and NEO populations.

In the case of the Koronis family (see, for example, Slivan 2003), some of its members have been forced into a narrow grouping of orientations, i.e., their spin axes are somewhat parallel. The careful modeling of spin axes for all asteroids and, in particular, families or groups with smaller members will allow refining the theories on the YORP effect.

Two recent example of how sunlight can affect asteroid rotation rates were presented by Kaasalainen et al (2007) and Lowry et al (2007). In both these papers, the authors gave proof that the YORP effect was indeed causing the asteroid spin rates to increase.

3. Modern Lightcurve Inversion

In 2001 Mikko Kaasalainen and his colleagues published two important papers that helped revolutionize the lightcurve inversion process. These two papers presented the problem as a matter of finding a convex shape that, when appropriate scattering laws were applied, would reproduce an asteroid's lightcurve at any given epoch.

The most important point is that the modeling process deals only with convex shapes and does not attempt to model concavities such as large craters or saddles directly. Instead, these areas appear as large flat areas in the final model. A good example is 433 Eros with its large saddle. In that case, the shape model shows a large flat area where the saddle would be.

For analogy, consider the model to be the shape of the "wrapping paper" put around the object with the paper stretching over the large concavities and not trying to fill them in. Kaasalainen showed that this approach not only produced good results but that the solutions were more stable.

Since these papers, many have followed using the algorithms involved so that models (spin axis and shape) are now available for more than 100 asteroids. What's more, those models for which there are direct comparisons against detailed occultation results or images from spacecraft or adaptive optics on large telescopes agree well within the given errors.

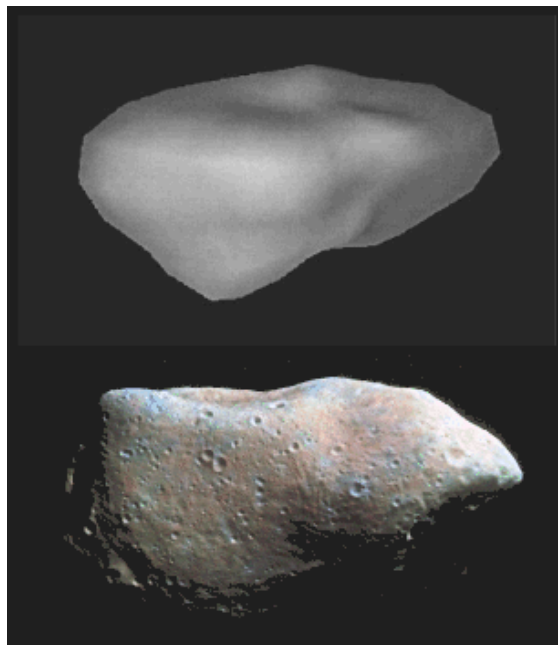


Figure 2. Comparison on model by Kaasalainen (top) and Galileo image of 953 Gaspra. Kaasalainen/NASA.

The rest of this section is based largely on the introductory material provided in the documentation for LCInvert from Bdw Publishing, but it is not specific to that program. Instead, it is a primer on asteroid modeling and can be applied when using programs based on the free open source code.

3.1. Free Modeling Source Code

The core libraries developed by Kaasalainen and Durech are available as free, open-source files written in C or Delphi, the latter using Delphi TObjects in place of large static arrays. The source code, including the aforementioned documentation, can be downloaded from

http://www.minorplanetobserver.com/MPOSoftware/Inversion_SourceCode.htm

Please observe the open source license agreement.

3. 2. Model with Care

It is very important to emphasize that lightcurve inversion is not a simple, straightforward process and is fraught with many more pitfalls than might ever be encountered when finding the period of a lightcurve. ***It is very easy to reach false conclusions.*** A certain amount of data is required that meets specific requirements.

To answer the most common question right off: You cannot get the shape or spin axis of a main-belt asteroid from data during a single apparition. It just cannot be done. An NEO that goes through a significant change in phase angle and viewing aspect during a single apparition might be an exception, but only if there is sufficient data.

For example, the model in Figure 3 was the result of doing a search and assuming the model with the best fit, i.e., the lowest ChiSq value, was a valid solution. In this case, all the ChiSq values were very similar, which is an almost sure indication that the available data cannot generate a good model.

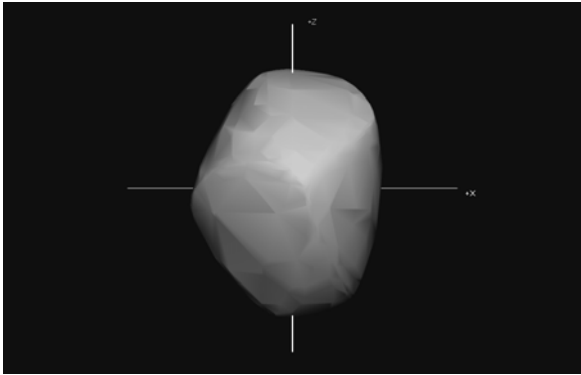


Figure 3. A bad shape model. The spin axis parallels the long axis of the asteroid.

3. 3. Relative versus Absolute Data

Absolute (or calibrated) data means that all the magnitudes (or intensities) have been put on a common system. This system can be internal or it can be one of the standard magnitude bands such as Johnson V. In addition, the data should be corrected to unity distance by applying, for magnitudes,

$$-5 \log(\Delta r)$$

Where Δ is the asteroid-Earth distance and r is the asteroid-Sun distance, in AU.

If data in different lightcurves are absolute (calibrated), they should not be shifted by arbitrary amounts within themselves to get the different curves to match. Supposedly, by virtue of being calibrated, they already do so.

In fact, you can take so-called "absolute data" (many times it is said to be but really isn't) and back out the unity distance correction if it was applied. The data set *can then be used as a relative data set*. This is done quite frequently since it is ***strongly*** recommended that only relative data sets be used when doing modeling.

Calibration is needed only for the sparse data sets anticipated from the Pan-STARRS and other surveys. In those cases, the observations will be made over many years and so the data need to be put on some sort of standard magnitude system.

Such calibration is sometimes required even during a single apparition. For example, if the asteroid has a long period, i.e., days or months. Getting the individual runs to align properly requires putting the data on a standard system, even if only internal. Again, once the entire data set is calibrated within itself, it is usually treated as a single relative set.

As you might gather from the above, a relative set of data has not formally been put on an internal or standard system. As often occurs with amateur (and even some professional work), data from several different nights are arbitrarily shifted to get a best fit. ***This is still relative data***, by virtue that the shifts are arbitrary. So, in general, amateurs are already providing the necessary lightcurve data required for modeling.

The ability to use relative data is what makes amateur work important and easy to incorporate within the planned efforts to use so-called sparse data sets generated by the large surveys to produce possibly thousands of shape and spin axis models in the first decade of the Pan-STARRS survey.

In short, the sparse data sets will generate good solutions in almost all cases. However, for refined solutions and to resolve unusual case (possible binary, etc.), dense lightcurves will be required. This is where the amateur community will fill in the gaps – by supplying just a few dense lightcurves of a given object. Of course, Pan-STARRS will reach much fainter than most amateurs, but there will still be more than enough work to occupy dozens observers for many years.

3. 4. Summary of Data Requirements

1. Lightcurves of sufficiently different geometries are required. (Which is why you can't get a good solution for a main-belt asteroid from one apparition since the viewing geometry is near the same for the whole apparition).

2. Lightcurves within the data set should cover a range of phase angles, including some at phase angles of $>10^\circ$ and, for an even better solution, $>20^\circ$.
3. The range of geometries and phase angle coverage is more important than the density of the data set, though excessively sparse lightcurves may be a hindrance.
4. Removing nearly redundant curves (in date) or bad (too sparse, high noise) can help remove biases and is more expedient than trying to apply weighting to individual curves.
5. The synodic period must be known to a sufficient degree of precision. The smallest separation ΔP of local minima in the period parameter space is roughly given by

$$\Delta P \approx 0.5 \frac{P^2}{\Delta t}$$

where Δt is the full epoch range of the data set. This derives from the fact that the maxima and minima of a double-sinusoidal lightcurve for periods P and $P \pm \Delta P$ are at the same epochs after Δt time.

The period and time span must be in the same units. The result is in those same units.

For a good solution, the initial period in the search should differ from the true sidereal period by no more than the ΔP given in the formula above. For example, assume the period is 6.0000 hr (0.25000 d) and that the observations span a range of five years, or about 1800 days. In this case, ΔP is about 0.0004 hr. If the span of data covers 30 years, about 11,000 days, then ΔP is about 0.000007 hr. If these levels of precision are not possible, then you should do a period scan based on the `period_scan C` or `Delphi` source code.

To put it in a nutshell, pole, period and the photometric convex hull of the object overwhelmingly dominate lightcurve morphology. Concavities and scatter properties are, in a way, comparable to noise: even though changing them does change the lightcurves somewhat in the direct problem, the solution of the inverse problem does not change much since there are no notably different convex shapes that could model the lightcurves better. The well-posedness and stability of the inversion procedure is thus a result of the convex formulation.

3. 5. Regularization Weight and Albedo Variations

One of the input parameters for the model search is a weighting factor given to the "dark facet". This facet is added as an absolute lightcurve (regardless of the data being used) that forces the model to be convex. *It has no effect on the lightcurve of the resulting model.*

The initial modeling process generates a file that includes the area and XYZ coordinates of the normal to the area. For a good result, the area of the dark facet must be less than 1% of the total area. If the area is too large, increasing the weight factor should decrease the size of the dark facet. This is particularly true if the Z-component of the normal of the dark facet is large, i.e., the normal is generally pointing up or down.

However, if the dark facet area remains large with increased weighting and it's normal is closer to right angles to the spin axis than not, this means there very likely a real albedo feature on the asteroid – a particularly bright or dark spot. This is very rare but does happen. In this case, increasing the weight doesn't fix the area problem and worsens the fit.

In this regard, use the ChiSq value of the fit for the model in comparison to that from other models to find the best model.

4. Sources for Modeling Data

Unfortunately, there is no great repository of asteroid lightcurve data such as there is for astrometry with the Minor Planet Center. The closest thing right now is the Standard Asteroid Photometry Catalog (SAPC) developed by Torppa et al

<http://www.astro.helsinki.fi/SAPC/index.jsp>

The example below shows a typical data set found on the SAPC site. It's important to note several critical fields in the header before you adapt the data for use in a modeling program.

First is "LT CORRECTED." The core libraries require that the Julian Date for the data be light-time corrected. If the given data are not corrected, that correction must be applied before modeling begins.

Next is "ABS PHOTOMETRY." If this is True, then the data has been converted to a standard system, be it the typical Johnson-Cousins, or some internal system. While the core libraries can handle combined lightcurve sets that contain absolute and relative data, it's best to put treat them all as relative data. As noted in the previous section, a calibrated (absolute) data set is often treated as being relative. The

```

*****
OBJECT      : 2, , Pallas
LIGHTCURVE   : 271
COLUMNS     : # Visual Error
LT CORRECTED : false
ABS PHOTOMETRY : false
*****
OBSERVING TIME : 2441780.585081      (Sun Apr 08 04:02:31 EET 1973)
LOCATION      : E 5.71 +43.93 Observatoire de Haute-Provence, France
REFERENCE   : Lustig and Hahn (1976)
INFORMATION :
  UPDATE.....: 0
  OBSERVING TIME: 1973 Apr 8.1
GEOMETRY:
  epoch      : 2441780.585081      (Sun Apr 08 04:02:31 EET 1973)
  Earth      : -0.949737225310019 -0.317430659536775 -1.99850829998031E-5
  object     : -2.05435606514992 -1.32150929163441 1.07575905867397
ASPECT DATA : 2.6691 1.8399 14.6 222.3 35.8 2441780.585081
DATA:
2441780.585081 8.192 0.0
2441780.586782 8.188 0.0
2441780.58853 8.182 0.0
2441780.590949 8.186 0.0
2441780.592373 8.189 0.0

```

Sample of Data from SAPP site

only disadvantage is that absolute data does allow fixing the "height" (Z-axis depth) of the asteroid while relative data does not, meaning a model using only relative data can find only the a/b ratio of the shape but not a/c or b/c.

Finally, there can be a "REDUCED" field. If not included, this implies the value is False. If the field is included and set to True, then the data have been set to unity distance by applying

$$-5 \log(Rr)$$

Where R is the asteroid's sun distance and r is the distance from Earth in AU.

If True, this correction must be removed if the data are being treated as relative and not absolute.

4. 1. Basic Data Requirements

In addition to the light-time corrected Julian Date and a magnitude, differential or "absolute" (not the same as absolute data), the input data files must contain the *asteroidcentric* XYZ ecliptic rectangular coordinates of the Sun and Earth.

Note that the Sun values are the negative of the heliocentric ecliptic coordinates of the asteroid. The Earth values are found by finding the vector sum of those coordinates and the geocentric ecliptic XYZ coordinates of the asteroid. The MPO LCInvert program computes these values and places them in the input data files automatically.

The free source code includes properly formatted sample files. The included documentation covers these requirements in more detail as well as provides a tutorial for preparing the data for LCInvert. The beginning part of that tutorial can be applied when creating data files for a different modeling program.

5. The Modeling Process

Once the data have been prepared, the modeling process begins. Even on today's fast desktop computers, do not expect to have instantaneous results. In some cases, it can take several hours for a single step within the process. However, the end result is what matters and doing it by hand is not an option.

5. 1. Finding the Period

As noted in Section 2, you must know the synodic rotation period of the asteroid to a very high degree of precision. Using a low precision period or one that's entirely wrong is a complete waste of computer time and, save only with "Lottery-Winning Luck", will produce completely wrong results.

The open source code contains a file that does the period search using the same data that you use for the initial modeling stage. It helps to have a good estimate of the period from other sources, such as Fourier analysis on the data set or previously published work. Note again that any of these sources must be both accurate and precise.

If using Fourier analysis on multiple lightcurves over several apparitions, remember that the synodic rate can change during a single apparition. Break the lightcurves into sets of about the same date and find as precise of periods as possible for each set. Use the average of those periods as the center point of your search and the spread of the values to dictate the range of periods to be searched.

5.2. Finding the Gaussian Shape Areas

The initial step of the modeling process starts with a near sphere cut into a number of rows with so many facets per row. The first step of the modeling process converts those facets from their original size and orientation to a set of areas and outward normals that duplicate the lightcurve data. Two forms of scattering are included in the modeling.

Several initial conditions must be set before starting this phase of modeling. This includes an initial estimate of the ecliptic longitude and latitude of the pole as well as the period. The last can be taken from the period search or other sources. If previous work has been published for the asteroid being studied, the longitude and latitude can be taken from there as well.

However, it may be a good idea to search for alternate solutions, especially if the inclination of the asteroid orbit is small, i.e., it is nearly on the same plane as the ecliptic. In this case, there can be an ambiguity in the solution where the true longitude is about 180° from the one found.

Quick mention will be made of one parameter, that is the size of the "dark area" that is required to assure that the final shape is a closed convex hull. See "Regularization Weight and Albedo Variations" above for a discussion about this value. It is critical to being able to find a proper model.

The others parameters will not be discussed in detail here. Instead, the reader is referred to the documentation included in the open source code libraries.

The MPO LCInvert program allows searching a grid of 15 possible longitude/latitude combinations and reports the ChiSq and area of the dark area. A grouping of the derived pole solution along with lowest ChiSq values gives a good sense of the quality of the solution. It would be a good idea for any modeling program to include such a grid search, especially for those times when there are no previously reported pole solutions available.

This process takes the least amount of time of all the steps with, usually, about 50 iteration steps being used. Sometime a higher value, e.g., 100, will pro-

vide a better model, but – of course – doing so extends the time to complete this step.

Naturally, if a grid search of pole solutions is implemented, then the time for this step is increased considerably. However, if such a search means finding a highly probable solution for final modeling, the extra time for this step is worth it.

5.3. The Minkowski Reduction

The previous step does not define a closed convex hull, only a set of areas and their directions. These must be put together, like a jigsaw puzzle, into a final closed form that fits the lightcurve data. This is done using the Minkowski source code file and can take a considerable amount of time. This is actually a very elegant process (only a mathematician would say that) and the reader is encouraged to read the two 2001 Kaasalainen papers to learn more about it.

The Minkowski method generates a file that contains two sets of data: the XYZ coordinates of the vertices of the set of polygons that form the final shape and the indices for the vertices that form each of the shapes.

Many 3-D modeling programs cannot handle polygonal shapes for creating a 3-D model but can use triangles. A simple routine in the source code libraries splits the Minkowski polygons into a, usually, larger number of triangles. These can then be used for 3-D rendering.

6. Modeling Results

The first attempts at modeling were to duplicate previously published work by Kaasalainen and others. Many subtleties were learned and some not so small mistakes made. However, there is no better teacher than experience and it was extremely gratifying to get the same results – eventually.

6. 1. 2 Pallas

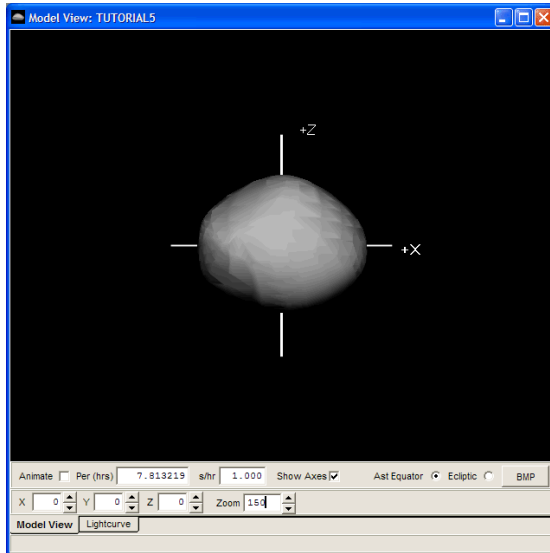


Figure 4. A model for 2 Pallas generated with LCInvert.

Torppa et al (2003) used data from the SAPC web site to determine the pole and shape for the second largest main-belt asteroid (or largest if Ceres is the only dwarf planet in the main belt and so not part of the group of small solar system bodies that includes asteroids). Their findings were:

```
Pole (long): 35°
Pole (lat): -12°
Period: 7.813225 hr
```

Using the same data set, my results were:

```
Pole (long): 35.3
Pole (lat): -15.2
Period: 7.81321904
```

Given its size, the asteroid is, as expected, somewhat spherical in shape.

6. 2. 43 Ariadne

Kaasalainen et al (2002) used data from the Uppsala Catalog (available via the SAPC site) to determine its model. Their data is part of the open source code distribution so it would be natural that my findings matched theirs. It was good to have that control since not matching the model when developing the open source Delphi code was an indication that something had been lost in translation.

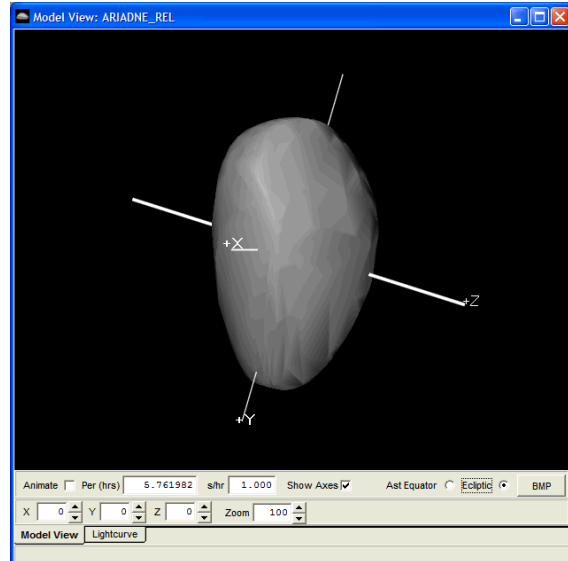


Figure 5. The model for 43 Ariadne as generated by LCInvert. The view is from the plane of the ecliptic looking towards the center of the asteroid. The sun is behind the observer.

Figure 5 shows the model as seen on the plane of the ecliptic. This shows the true orientation of the spin axis and the sense of rotation, which is always counter clockwise as viewed from above the north pole. Figure 7 shows the view from above the asteroid's equator at local noon at its "prime meridian."

```
Pole (long/1253°)
Pole (lat): -15°
Period: 5.761986 hr
```

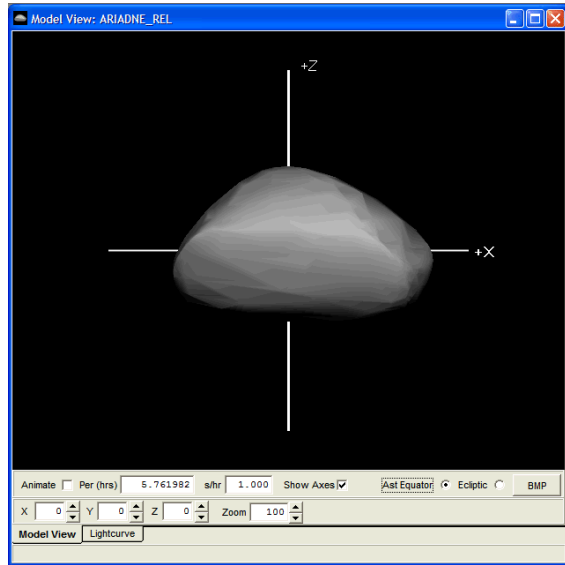


Figure 6. The model for 43 Ariadne as generated by LCInvert. The view is from above the subsolar point on the asteroid's equator at 0° rotation ("prime meridian").

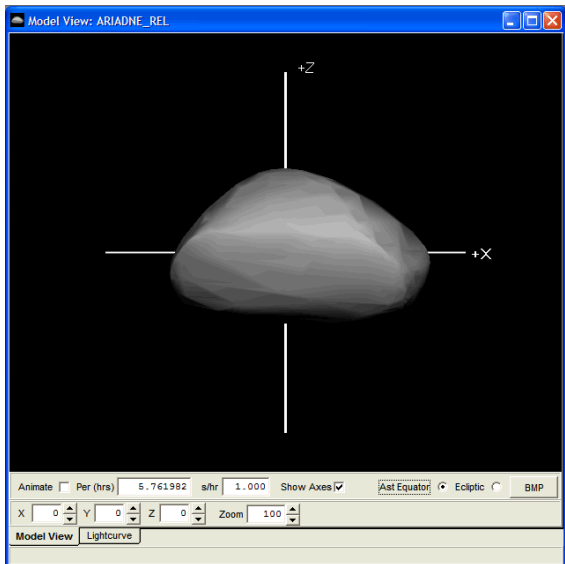


Figure 7. The model for 43 Ariadne as generated by LCInvert. The view is from above the subsolar point on the asteroid's equator at 0° rotation ("prime meridian").

7. 1600 Vyssotsky

Once two known models were re-created, the next test was to work with data on an asteroid that was not previously modeled. This turned out to be the Hungaria asteroid, 1600 Vyssotsky. Lightcurves covering three apparitions were obtained by the author and other observers (Warner 1999, Warner 2006).

The viewing aspects were significantly different in one apparition, which led to hopes that a good model could be obtained. Unfortunately, that was not the case. The ChiSq values for the various solutions did not show a clear-cut solution. This was supported by the fact that the 15-position grid search did not show any substantial grouping of pole solutions.

The two best models (lowest ChiSq) had the following solutions:

Longitude: 113.7
 Latitude: +14.5
 Sidereal Period: 3.19997896

Shown in Figure 8

Longitude: 272.6
 Latitude: -46.9
 Sidereal Period: 3.19997909

Shown in Figure 9. The two periods are essentially identical but the pole solutions are dramatically different.

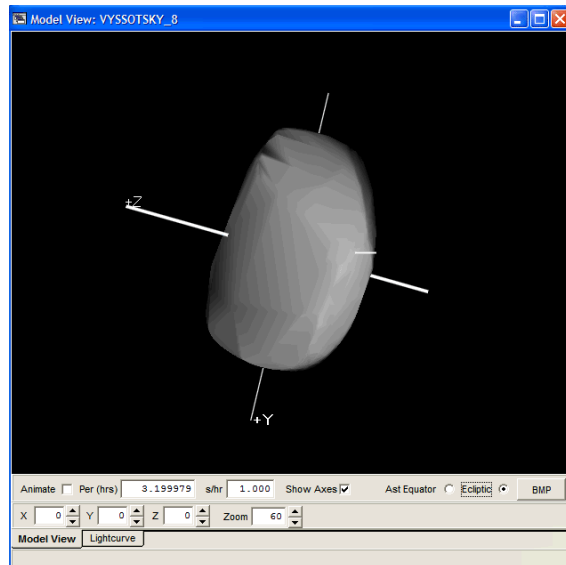


Figure 8. A model for 1600 Vyssotsky using the ecliptic view, which is from the plane of the ecliptic looking towards the center of the asteroid. The sun is behind the observer.

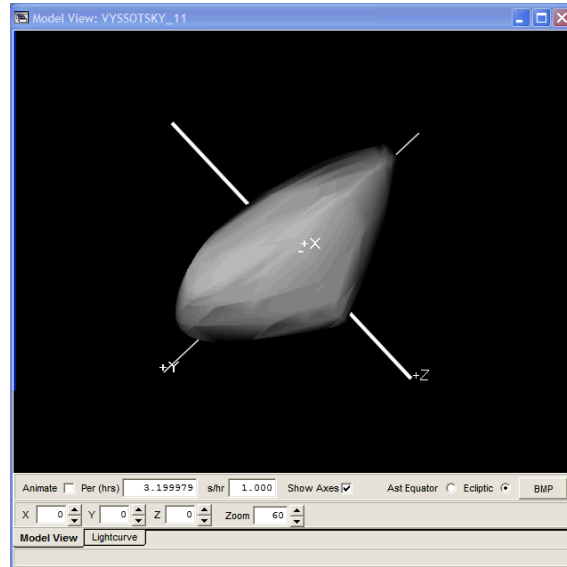


Figure 9. An alternate model for 1600 Vysotsky. There were insufficient data to find a consistent model.

One way to check the solution is to fit a lightcurve from the model to the actual data. Figure 10 shows such a comparison. As you can see, the actual and theoretical curves fit very well. However, this alone does not mean that the correct model was found.

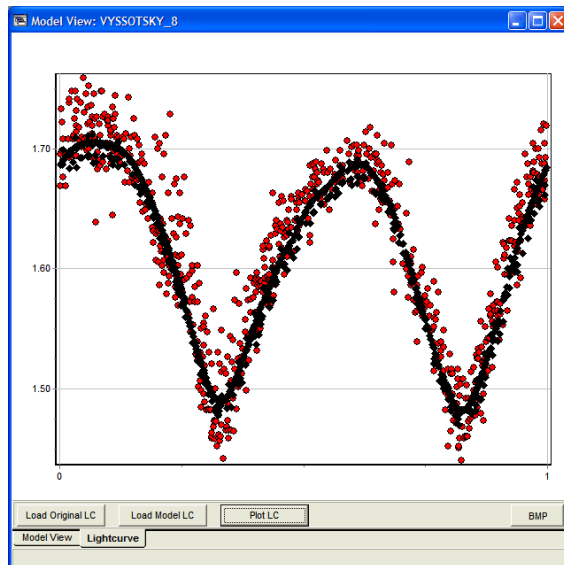


Figure 10. Comparing the model lightcurve to the original data. The red dots represent the original data while the black dots represent the model. The November 2005 data produced nearly identical comparison plots with the theoretical model.

Both models shown above produced nearly identical comparison plots. The ChiSq value for neither model was significantly different from the other. This

is the better test, i.e., that one model have a ChiSq solution that is at least 10-20% better than any other solution.

8. Conclusion

While working asteroid lightcurve inversions is not a simple process and yields "bad" results more often than good results, it is no longer strictly for professional astronomers. There are many asteroids wanting only another apparition or two before a model can be found. Amateurs have access to most of the data available, and so they are not shut out from this important field of research.

Furthermore, with the large surveys such as Pan-STARRS coming on line, the work for amateurs will be even more important. It is they who can provide the dense lightcurves, concentrated data runs that are used in conjunction with the sparse data sets provided by the surveys that will produce the best possible results. The sparse data sets can find good models for thousands of objects but they cannot resolve questionable results. That will be the role of the amateur.

The large surveys may be limited in how bright they can go, leaving open for those with more modest equipment, still hundreds of asteroids where amateur work can dominate.

The collection of data is vital to the advancement of asteroid research. However, analysis is just as important and, for many, provides the extra incentive that compensates for the sometimes drudgery of taking and measuring images. Being able to participate fully in both sides of scientific endeavor allows the amateur to become not just a participant but a significant and vital collaborator.

9. Acknowledgements

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